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Conjugate heat/mass transfer from a circular cylinder with an internal heat/mass source in laminar crossflow at low Reynolds numbers

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Abstract

This paper presents a theoretical study of conjugate heat/mass transfer from a circular cylinder with an internal heat/mass source and a surrounding fluid flow. The heat/mass source consists of a constant temperature/concentration wire imbedded in the cylinder center. A finite difference method discretizes the equations. The multigrid method solves the discrete system. Numerical investigations were carried out for cylinder *Re* numbers equal to 2 and 20. The values of the *Pr* number were selected such that the product $Re \times Pr$ is constant and equal to 100. The main aspect analysed is the influence of the conductivity ratio on the local and average *Nu* numbers at different values of the wire diameter.

1. Introduction

Two recent papers [1,2], analyse against different frameworks and with different aims the conjugate heat transfer from a circular cylinder with an internal heat source. Lin et al. [1] study the inverse analysis to determine the thermal boundary behaviour of a heated cylinder normal to a laminar air stream. The mathematical model consists of the conduction equation for the temperature field inside the cylinder. The hot wire imbedded in the center of the heated cylinder is assumed to be a point source of constant temperature. At the interface between the cylinder and the fluid, the boundary condition

$$\lambda_c \frac{\partial T_c}{\partial r} = h(\theta)(T_c - T_\infty)$$

holds. The local heat transfer coefficient $h(\theta)$ is taken from [3]. The forced convection heat transfer from a circular cylinder with constant temperature was studied in [3].

Haldar [2] evaluates the heat losses from a horizontal cylinder maintained at a constant temperature and covered with a layer of insulation. For the insulation layer, the mathematical model reduces to the conduction equation. In the environmental fluid, buoyancy generates free convection motion around the insulation. The mathematical model of the fluid region consists of the coupled Navier–Stokes-energy equations. Continuity of the heat flux is assumed at the insulation–fluid interface.

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Nomenclature

C	concentration of the transferring species
d	cylinder diameter
D	diffusion coefficient of the transferring species
H	Henry number
Pr	fluid phase Prandtl (Schmidt) number, $Pr = \nu/\alpha_f(D_f)$
Re	Reynolds number based on cylinder diameter, $Re = U_\infty d/\nu$
T	temperature
Z	dimensionless variable, $Z_{c(f)} = \frac{T_{c(f)}-T_\infty}{T_0-T_\infty}$ or $Z_c = \frac{C_c-C_\infty H}{C_0-C_\infty H}$, $Z_f = \frac{C_f-C_\infty}{C_0/H-C_\infty}$
α	thermal diffusivity
Φ	λ_c/λ_f or (D_c/D_f)/H
λ	thermal conductivity
ν	kinematic viscosity of the fluid phase

Subscripts

c	refers to cylinder
f	refers to environmental fluid
0	refers to the heat/mass source
∞	large distance from the cylinder

This paper wishes to complete [1,2] by solving the conjugate forced convection heat/mass transfer from a circular cylinder with an internal heat/mass source in steady, laminar crossflow. The internal source consists of an inner cylinder maintained at constant temperature/concentration. Industrial and environmental processes in which this phenomenon plays an important role are: cooling of different materials, insulating processes, anemometry and chemical or radioactive contamination/purification. Based on the arguments of [4], i.e. in the regime corresponding to the transition from a 2-D steady to a 2-D periodic wake the heat was never a passive contaminant for the flow field, we restricted the present study to two hydrodynamic regimes: steady flow without separation ($Re = 2$) and steady flow with two symmetric vortices behind the cylinder ($Re = 20$). The main aspect analysed is the influence of the conductivity ratio and inner cylinder diameter on the heat transfer rate.

2. Basic equations

Let us consider an infinitely long horizontal circular cylinder of diameter d (considerably higher than the molecular mean free path of the surrounding fluid) placed in a vertical, laminar, steady flow of an incompressible Newtonian fluid having a free stream velocity U_∞ and constant temperature/concentration, T_∞/C_∞. A wire of dimensionless radius r_0 and constant temperature/concentration T_0/C_0 is inserted inside the cylinder. During the heat/mass transfer there is no phase change and chemical reaction. The presence of the pressure diffusion or thermal diffusion is ignored. The effects of radiative heat transfer, buoyancy and viscous dissipation are considered negligible. Assuming also constant physical properties of the cylinder and surrounding medium, the governing nondimensional equations, in cylindrical coordinates (r, θ) are:

– inside the cylinder

$$(r \leqslant r_0), Z_c = 1.0; \ (r_0 < r < 1), \ \frac{\partial^2 Z_c}{\partial r^2} + \frac{1}{r}\frac{\partial Z_c}{\partial r} + \frac{1}{r^2}\frac{\partial^2 Z_c}{\partial \theta^2} = 0.0 \tag{1}$$

– in the surrounding fluid

$$\frac{Re\,Pr}{2}\left(V_R\frac{\partial Z_f}{\partial x} + \frac{V_\theta}{r}\frac{\partial Z_f}{\partial \theta}\right) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial Z_f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 Z_f}{\partial \theta^2} \tag{2}$$

The boundary conditions to be satisfied are:

– interface ($r = 1$);

$$Z_c = Z_f, \Phi\frac{\partial Z_c}{\partial r} = \frac{\partial Z_f}{\partial r} \tag{3a}$$

– free stream ($r = \infty$);

$$Z_f = 0.0 \tag{3b}$$

– symmetry axis ($\theta = 0, \pi$);

$$\frac{\partial Z_c}{\partial \theta} = \frac{\partial Z_f}{\partial \theta} = 0.0 \tag{3c}$$

The assumptions practiced here are typically for the analysis of the analogy between heat and mass transfer. Under these conditions, the mathematical model (1–3) describes concomitantly a mass/heat transfer process and the dimensionless variable Z has a double signification, dimensionless concentration/dimensionless temperature.

The components of the velocity field (V_R, V_θ) were calculated by solving the Navier–Stokes equations. Details about hydrodynamic computations are presented in [5].

The heat/mass exchange between the cylinder and the fluid is measured by means of the local and/or mean (surface-average) heat/mass transfer coefficients. Considering as driving force the difference $(T_0 - T_\infty)$ or

($C_0 - C_\infty$), the local Nusselt number based on the diameter of the cylinder is

$$Nu(\theta) = -2\frac{\partial Z_c}{\partial r}\bigg|_{r=1-} \quad \text{if } \Phi \leqslant 1;$$
$$Nu(\theta) = -2\frac{\partial Z_f}{\partial r}\bigg|_{r=1+} \quad \text{if } \Phi > 1 \tag{4}$$

The subscripts 1− and 1+ refer to the left and right derivative, respectively. The surface-average overall Nu number was calculated with the relation:

$$Nu = \frac{1}{\pi}\int_0^\pi Nu(\theta)\,d\theta \tag{5}$$

The surface-average fractional Nu numbers, i.e. the internal Nu number, Nu_{int}, and the external Nu number, Nu_{ext}, were computed as

$$Nu_{int} = -\frac{2}{\pi(1.0-\overline{Z}_{c,s})}\int_0^\pi \frac{\partial Z_c}{\partial r}\bigg|_{r=1-} d\theta,$$
$$Nu_{ext} = -\frac{2}{\pi\overline{Z}_{c,s}}\int_0^\pi \frac{\partial Z_f}{\partial r}\bigg|_{r=1+} d\theta \tag{6}$$

where $\overline{Z}_{c,s}$ is the dimensionless surface-average temperature/concentration

$$\overline{Z}_{c,s} = \frac{2}{\pi}\int_0^\pi Z_c|_{r=1}\,d\theta \tag{7}$$

The relation between the fractional and overall Nu numbers is

$$\frac{1}{Nu} = \left(\frac{1}{Nu_{int}} + \Phi\frac{1}{Nu_{ext}}\right) \quad \text{if } \Phi \leqslant 1;$$
$$\frac{1}{Nu} = \left(\frac{1}{\Phi}\frac{1}{Nu_{int}} + \frac{1}{Nu_{ext}}\right) \quad \text{if } \Phi > 1 \tag{8}$$

3. Method of solution

The energy/mass balance equations were solved numerically. The radial coordinate r for the outer region was replaced by x through the transformation $r = \exp x$. The external boundary condition is assumed to be valid at a large but finite distance, r_∞, from the center of the cylinder. For the Re and Pr values employed in this study, $r_\infty = \exp(\pi)$ is a good approximation.

Eq. (1) was discretized with the central second order accurate finite difference scheme. In the fluid phase, the exponentially fitted scheme [6], was used. The discrete elliptic equations were solved with the multigrid (MG) [7,8] algorithm (the classical FAS scheme). The structure of the MG iteration step (cycle) is: (1) cycle of type V; (2) smoothing by alternating line Gauss–Seidel; (3) the discrete operator is used for the residual restriction and for transferring the coarse grid corrections back to finer grids; 4) the solution is transferred on coarser grid by injection. Four levels were used. The coarsest mesh has 17×17 or 33×33 points in each phase. The finest grid has 129×129 or 257×257 points in each phase.

4. Results

The key parameters in any conjugate problem are those that characterize the coupling features of the transfer at the interface. In this work there is only one parameter, denoted by Φ, related to the transfer at the interface. In the case of a heat transfer process Φ represents the ratio (cylinder/surrounding medium) of the thermal conductivities while in the case of a mass transfer process Φ is the ratio (diffusivities ratio)/(Henry number). To meet the situations of practical interest and to study the asymptotic behaviour, Φ takes values in the range 10^{-2}–10^2. The influence of Φ on the local and average Nu numbers was studied for the following r_0 values: 0.0 (as in [1]), 0.1, 0.25, 0.50, 0.75, 0.90, 0.95 and 1.0. The case $r_0 = 1.0$ corresponds to the heat/mass transfer from a cylinder with constant temperature/concentration.

The variation of Φ influences or not Pr. To have a clear picture about the effect of Φ on the transfer rate, Pr was kept constant. To avoid a disturbing increase in the numerical errors, Pr was selected such that the product $Re\,Pr$ is equal to 100 for both Re values. The criteria used to choose Re were discussed in Section 1.

Unfortunately, there are no data in literature to verify the accuracy of the present computations. Based on the experience of the sphere problem and [5], one may expect that, when Φ tends to infinity, Nu tends to the value corresponding to the heat/mass transfer from a cylinder with constant temperature/concentration. Forced convection heat/mass transfer from a circular cylinder with constant temperature/concentration in laminar crossflow have been studied in [3,9–15]. For the same problem, many correlation formulae used to calculate Nu could be found in [16].

The present Nu values obtained at $r_0 = 1$ and those provided by published predictive relations can be viewed in Table 1. Table 1 shows a good agreement between the present and published results. The comparison of the present conjugate solutions at $\Phi \gg 1$ with the solutions obtained at $r_0 = 1$ is presented in the next paragraphs.

Table 1
Comparison of overall Nu values computed at $r_0 = 1$ with previous studies

Re	Pr	Nu	Authors
2	50	3.6314	Kurdyumov and Fernandez [14]
		3.88813	Kramers (Dennis et al. [9])
		3.593	Present
20	5	4.9384	Kramers (Dennis et al. [9])
		4.596	Present

The influence of Φ on the local Nu number, $Nu(\theta)$, at different r_0 values, is presented in Fig. 1 ($Re = 2, Pr = 50$) and Fig. 2 ($Re = 20, Pr = 5$). Obviously, the results plotted depend on the relations used to calculate local Nu. However, we will insist on the aspects independent from the relation that defines $Nu(\theta)$. Figs. 1 and 2 show that both Φ and r_0 have a strong influence on local Nu. This influence follows the same rules at $Re = 2, Pr = 50$ and $Re = 20$, $Pr = 5$. For $\Phi \to \infty$ and independent on r_0 values ($r_0 < 1$), $Nu(\theta)$ tends to the values obtained at $r_0 = 1$. Φ does not affect only the $Nu(\theta)$ values. The dependence of the local Nu number versus θ depends also on Φ. At very small values of Φ, $\Phi \to\ 0$, $Nu(\theta)$ does not depend practically on θ. The increase in Φ increases the angular dependence of $Nu(\theta)$. The influence of r_0 on $Nu(\theta)$ is significant especially for $\Phi \leqslant 1$. As expected, the increase in r_0 increases $Nu(\theta)$. We must also mention that the mass/thermal wake phenomenon [5] is not present.

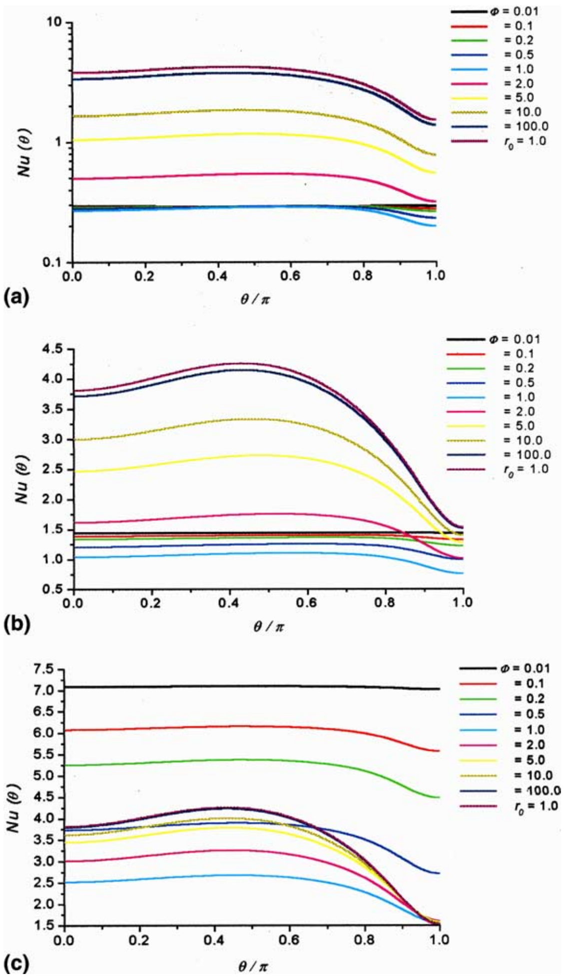

Fig. 1. The effect of Φ on the local Nu numbers at $Re = 2$ and $Re\,Pr = 100$; (a) $r_0 = 0.0$; (b) $r_0 = 0.250$; (c) $r_0 = 0.750$.

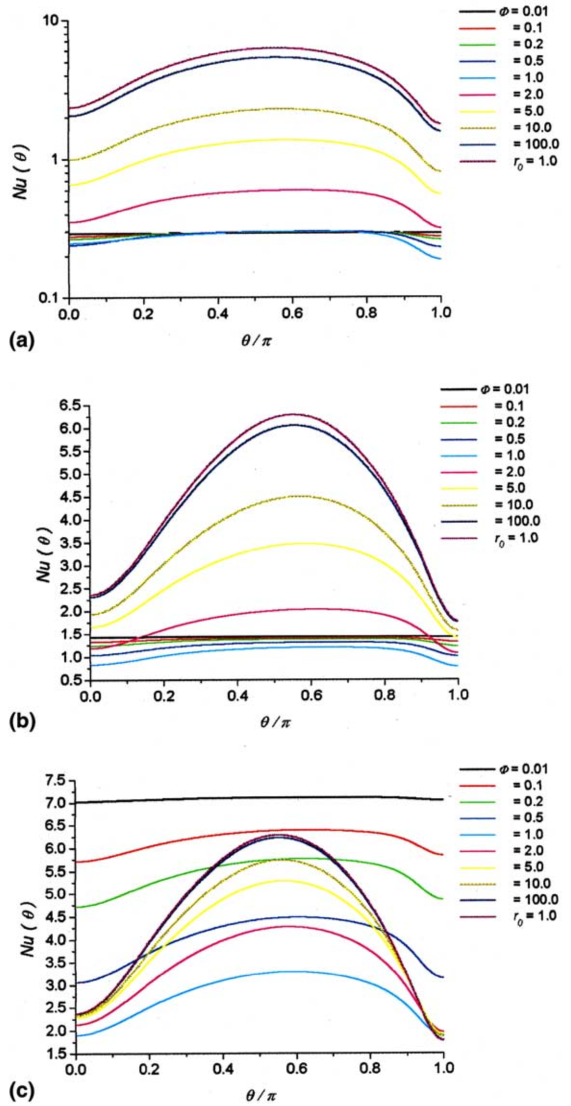

Fig. 2. The effect of Φ on the local Nu numbers at $Re = 20$ and $Re\,Pr = 100$; (a) $r_0 = 0.0$; (b) $r_0 = 0.250$; (c) $r_0 = 0.750$.

The influence of Φ on the fractional Nu numbers is completely different from that encountered in [5]. The behaviour of the fractional Nu numbers in the parameter space (Φ, r_0) can be summarized by the following statements:

– Nu_{ext} does not depend on r_0; Φ influences slightly Nu_{ext} (see Fig. 3); this influence depend on Re; at $Re = 2$ the increase in Φ decreases Nu_{ext} while at $Re = 20$ the increase in Φ increases Nu_{ext}; the relative difference between Nu_{ext} $(\Phi = 0.01)$ and Nu_{ext} $(\Phi = 100)$ is approximately 3% at $Re = 2$ and 6.6% at $Re = 20$; for $\Phi \to \infty$, Nu_{ext} tends to the values pro-

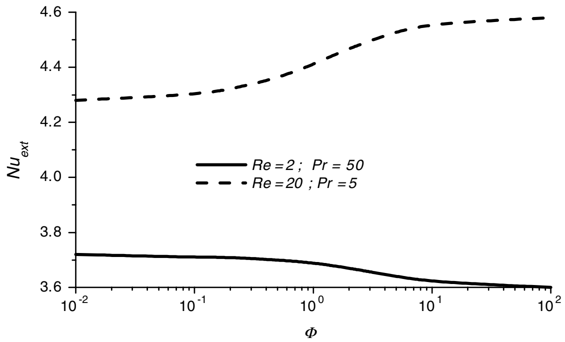

Fig. 3. The influence of Φ on the external Nu number.

vided by the case $r_0 = 1$ (at $\Phi = 100$, the relative difference is smaller than 1%).

– Nu_{int} does not depend on Φ, Re and Pr; Nu_{int} depends only on r_0 (see Fig. 4 and Table 2); we tried to approximate the dependence Nu_{int} versus r_0 by a formula but the results were unsatisfactory.

The effect of Φ variation on overall Nu is that expected. When $\Phi \to 0$, Nu tends to Nu_{int}, while if $\Phi \to \infty Nu$ tends to Nu_{ext} (see Fig. 5). The dependence Nu versus Φ has a minimum at $\Phi = 1$. The increase in r_0 increases Nu at moderate and small values of Φ.

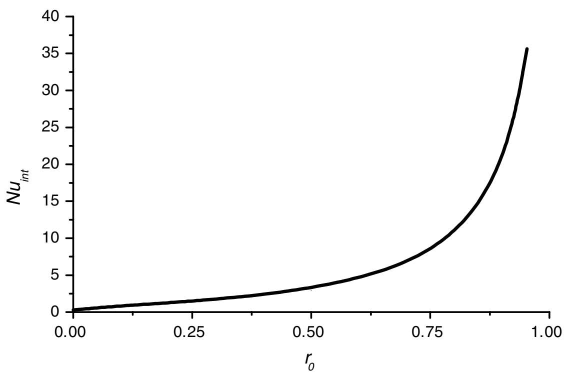

Fig. 4. The dependence Nu_{int} versus r_0.

Table 2
Nu_{int} at different r_0 values

R_0	Nu_{int}
0.0	0.27
0.10	0.86
0.25	1.45
0.5	2.91
0.75	7.20
0.9	18.7
0.95	35.6

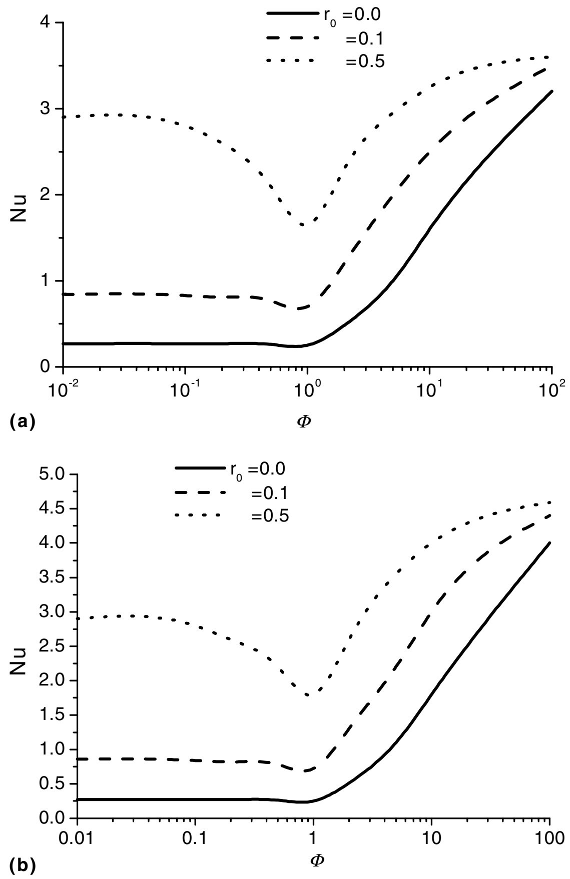

Fig. 5. The influence of Φ on the overall Nu number: (a) $Re = 2$ and $RePr = 100$; (b) $Re = 20$ and $RePr = 100$.

Regarding the overall Nu number we also studied the following problem: the accuracy of the approximation obtained by replacing in (8) Nu_{ext} by the value provided by the case $r_0 = 1$. The answer to this problem is positive. The relative error between the approximate overall Nu and the present numerical results does not overflow 1%.

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